

(2/2/3)-SAT problem and its applications in dominating set problems

Arash Ahadi^a, Ali Dehghan^b,

^a*Department of Mathematical Sciences, Sharif University of Technology, Tehran, Iran*

^b*Systems and Computer Engineering Department, Carleton University, Ottawa, Canada* *

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Abstract

The satisfiability problem is known to be **NP**-complete in general and for many restricted cases. One way to restrict instances of k -SAT is to limit the number of times a variable can occur. It was shown that for an instance of 4-SAT with the property that every literal appears in exactly 4 clauses (2 times negated and 2 times not negated), determining whether there is an assignment for literals, such that every clause contains exactly 2 true literals and 2 false literals is **NP**-complete. In this work, we show that deciding the satisfiability of 3-SAT with the property that every literal appears in exactly 4 clauses (2 time negated and 2 times not negated), is **NP**-complete. We call this problem (2/2/3)-SAT. For a nonempty regular graph $G = (V, E)$, it was asked by [1] to determine whether for a given independent set T , there is an independent dominating set D for T such that $T \cap D = \emptyset$? As an application of (2/2/3)-SAT problem, we show that for every $r \geq 3$, this problem is **NP**-complete.

It is well-known that the vertex set of every graph without isolated vertices can be partitioned into two dominating sets [10]. Determining the computational complexity of deciding whether the vertices of a given connected cubic graph G can be partitioned into independent dominating sets remains unsolved. We show that this problem is **NP**-complete, even if restricted to (i) connected graphs with only two numbers in their degree sets, (ii) cubic graphs. Finally, we study the relationship between 1-perfect codes and the incidence coloring of graphs and as another application of our complexity results, we prove that for a given cubic graph G deciding whether G is 4-incidence colorable is **NP**-complete.

* *E-mail addresses:* arash_ahadi@mehr.sharif.edu (Arash Ahadi), ali_dehghan16@aut.ac.ir, alidehghan@sce.carleton.ca (Ali Dehghan).

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1 Introduction and Results

The satisfiability problem is known to be **NP**-complete in general and for many restricted cases. Finding the strongest possible restrictions under which the satisfiability problem remains **NP**-complete is important since this can make it easier to prove the **NP**-completeness of new problems by allowing easier reductions. An instance of k -SAT is a set of clauses that are disjunctions of exactly k literals. The problem is to determine whether there is an assignment of truth values to the variables such that all the clauses are satisfied. One way to restrict instances of k -SAT is to limit the number of times a variable can occur. Consider a 4-SAT formula with the property that each clause contains 4 literals and each variable appears 4 times in the formula, twice negated and twice not negated; determining whether there is a truth setting for the formula such that in each clause there are exactly 2 true literals is **NP**-complete. In this work, we show that a similar version of this problem is **NP**-complete [11].

For a formula $\Phi = (X, C)$, a truth assignment is a mapping which assigns to each variable one of the two values *true* or *false*. A truth assignment satisfies a clause $c \in C$ if c contains at least one literal whose value is *true*. A truth assignment satisfies a CNF formula (a Boolean formula in Conjunctive Normal Form) if it satisfies each of its clauses. Given a CNF formula F , the satisfiability problem asks to determine if there is a truth assignment satisfying F . We show that deciding the satisfiability of 3-SAT with the property that every literal appears in exactly 4 clauses (2 time negated and 2 times not negated), is **NP**-complete. We call this problem (2/2/3)-SAT. Note that if we consider 3-SAT problem with the property that every literal appears in 2 clauses (1 time positive and 1 time negative), then the problem is always satisfiable. Also, Tovey in [13] showed that instances of 3-SAT in which every variable occurs three times are always satisfiable (this is an immediate corollary of Hall's Theorem). Therefore our result is sharp. In other words, in terms of (k, t, t') it is the tightest possible restriction in the sense that k -SAT remains **NP**-complete where each variable appears t time positive and t' time negative. Note that it is well known that 2-SAT can be solved in polynomial time.

Also, note that the following similar restriction is mentioned in the book Computational Complexity by Papadimitriou (page 183): Allowing clauses of size 2 and 3, with each variable appearing 3 times, and each literal at most 2, is **NP**-complete (but if all clauses have size 3 it is in **P**). This is a nice dichotomy that immediately gives the guess that the

(2/2/3)-SAT version shown here is **NP**-complete.

Suppose that $G = (V, E)$ is a graph and let $D, T \subseteq V(G)$. We say D is a dominating set for T , if for every vertex $v \in T$, we have $v \in D$ or there is a vertex $u \in D$ such that $vu \in E(G)$. For a nonempty regular graph $G = (V, E)$, it was asked by [1] to determine whether for a given independent set T , there is an independent dominating set D for T such that $T \cap D = \emptyset$? As an application of (2/2/3)-SAT problem, we show that for every $r \geq 3$, this problem is **NP**-complete.

Theorem 1

- (i) (2/2/3)-SAT problem is **NP**-complete.
- (ii) Let $r \geq 3$ be a fixed integer. Given (G, T) , where G is an r -regular graph and T is a maximal independent set of G , it is **NP**-complete to determine whether there is an independent dominating set D for T such that $T \cap D = \emptyset$.

The vertex set of every graph without isolated vertices can be partitioned into two dominating sets [10]. For any $k \geq 3$, Heggernes and Telle showed that it is **NP**-complete to determine whether a graph can be partitioned into k independent dominating sets [5]. It was shown that it is **NP**-hard to determine the chromatic index of an k -regular graph for any $k \geq 3$ [7]. Heggernes and Telle reduced this problem to their problem. For a given k -regular graph G they construct a graph \mathcal{J}_k such that the chromatic index of G is k if and only if the vertices of \mathcal{J}_k can be partitioned into k independent dominating sets. We show that always \mathcal{J}_k can be partitioned into $k + 1$ independent dominating sets. Determining the computational complexity of deciding whether the vertices of a given connected cubic graph G can be partitioned into independent dominating sets is unsolved and has a lot of applications in proving the **NP**-hardness results for other problems. Here, we focus on this problem and present an application.

We show that deciding whether the vertices of a given graph G can be partitioned into independent dominating sets is **NP**-complete, even for restricted class of graphs.

Theorem 2

- (i) The vertices of \mathcal{J}_k can be partitioned into $k + 1$ independent dominating sets.
- (ii) For a given connected graph G with at most two numbers in its degree set, determining whether the vertices of G can be partitioned into independent dominating sets is **NP**-complete.
- (iii) Determine whether the vertices of a given 3-regular graph can be partitioned into independent dominating sets is **NP**-complete.

There is a close relationship between 1-perfect codes and the incidence coloring of graphs. We will use this relationship and prove a new complexity result for the incidence coloring of cubic graphs. An *incidence* of a graph G is a pair (v, e) with $v \in V(G)$, $e \in E(G)$, such that v and e are incident. Two distinct incidences (v, e) and (w, f) are adjacent if one of the following holds:

- (i) $v = w$, or
- (ii) v and w are adjacent and $vw \in \{e, f\}$.

An *incidence coloring* of a graph G is a mapping from the set of incidences to a color set such that adjacent incidences of G are assigned distinct colors. The *incidence chromatic number* is the minimum number of colors needed and denoted by $\chi_i(G)$.

The concept of incidence coloring was first introduced by Brualdi and Massey in 1993 [2]. They said that determining the incidence chromatic number of a given cubic graph is an interesting question. After that the incidence coloring of cubic graphs were investigated by several authors [8, 9, 12]. In 2005 Maydanskiy proved that Incidence Coloring Conjecture holds for any graph with $\Delta(G) \leq 3$ [9]. Therefore, for a given cubic graph G , $\Delta(G) + 1 \leq \chi_i(G) \leq \Delta(G) + 2$. For a graph G with $\Delta(G) = 3$, if the degree of any vertex of G is 1 or 3, then the graph G is called a semi-cubic graph. In 2008, it was shown that it is **NP**-complete to determine if a semi-cubic graph is 4-incidence colorable [8]. Here, we improve the previous complexity result and show the following:

Theorem 3 *For a given 3-regular graph G deciding whether G is 4-incidence colorable is **NP**-complete.*

We follow [4, 15] for terminology and notations are not defined here, and we denote $\{1, 2, \dots, n\}$ by $[n]$. We denote the vertex set and the edge set of G by $V(G)$ and $E(G)$, respectively. The maximum degree and minimum degree of G are denoted by $\Delta(G)$ and $\delta(G)$. Also, for every $v \in V(G)$ and $X \subseteq V(G)$, $d(v)$, $N(v)$ and $N(X)$ denote the degree of v , the neighbor set of v and the set of vertices of G which has a neighbor in X , respectively. We say that a set of vertices are *independent* if there is no edge between these vertices. The *independence number*, $\alpha(G)$, of a graph G is the size of a largest independent set of G . A *clique* in a graph $G = (V, E)$ is a subset of its vertices such that every two vertices in the subset are connected by an edge. A *dominating set* of a graph G is a subset D of $V(G)$ such that every vertex not in D is joined to at least one vertex of D . For $k \in \mathbb{N}$, a *proper edge k -coloring* of G is a function $c : E(G) \rightarrow [k]$, such that if $e, e' \in E(G)$ share a common endpoint, then $c(e)$ and $c(e')$ are different. The smallest integer k such that G has a proper edge k -coloring is called the *chromatic index* of G and denoted by $\chi'(G)$. By Vizing's theorem [14], the chromatic index of a graph G is equal to either $\Delta(G)$ or $\Delta(G) + 1$.

2 Proofs

Proof of Theorem 1. It was shown that 2-in-(2/2/4)-SAT is **NP**-complete [11]. This version of SAT is defined as follows:

Each clause contains four literals; each variable appears four times in the formula, twice negated and twice not negated; the question is whether there is a truth setting for the formula such that in each clause there are exactly two true literals.

We prove the two parts of the theorem together. Assume that $r \geq 3$ is a fixed integer. Let Φ be a given formula with the set of variables X and the set of clauses C . We transform the formula Φ into a formula Φ''' such that in Φ''' each variable appears four times in the formula, twice negated and twice not negated. Also, Φ''' has a satisfying assignment if and only if Φ has a satisfying assignment such that in each clause there are exactly two true literals. Next, we transform the formula Φ''' into an r -regular graph G' with a maximal independent set T such that the graph G' has an independent dominating set D for T if and only if the formula Φ''' has a satisfying assignment. Our proof consists of five steps.

Step 1.

For every clause $c = (\alpha \vee \beta \vee \gamma \vee \zeta)$ in Φ , consider the ten clauses $(\star\alpha \vee \star\beta \vee \star\gamma \vee \star\zeta)$ in Φ' (for each variable α , $\star\alpha$ means one of α or $\neg\alpha$) such that the number of negative literals in each of the clauses in Φ' is not exactly 2. In other words, for instance $(\neg\alpha \vee \neg\beta \vee \gamma \vee \neg\zeta)$ and $(\alpha \vee \beta \vee \gamma \vee \zeta)$ are in Φ' , but $(\neg\alpha \vee \beta \vee \gamma \vee \neg\zeta)$ is not in Φ' .

Since in Φ each variable appears four times, twice negated and twice not negated. In Φ' every literal appears 20 times (each variable appears 40 times). Also, Φ has a satisfying assignment such that in every clause there are exactly two true literals if and only if Φ' has a satisfying assignment (there is at least one true literal in each clause).

Step 2.

For every clause $c = (\alpha \vee \beta \vee \gamma \vee \zeta)$ in Φ' , consider two new variables a_c, b_c and the following four clauses in Φ'' :

$$(\alpha \vee a_c \vee b_c), (\beta \vee a_c \vee \neg b_c), (\gamma \vee \neg a_c \vee b_c), (\zeta \vee \neg a_c \vee \neg b_c).$$

It is easy to see that the formula Φ' has a satisfying assignment if and only if the formula Φ'' has a satisfying assignment.

In Φ'' some of the variables appear 40 times. Call them old variables. For each old variable x , consider the new variables $x_0, x_1, \dots, x_{19}, y_0, \dots, y_{19}$ and for every $i, i \in \mathbb{Z}_{20}$, put the following clause in Φ'' :

$$\begin{cases} (x_i \vee y_i \vee y_i), (\neg x_i \vee \neg y_{i-1} \vee \neg y_{i-1}), & \text{if } i \text{ is odd} \\ (x_i \vee \neg y_{i-1} \vee \neg y_{i-1}), (\neg x_i \vee y_i \vee y_i) & \text{otherwise.} \end{cases}$$

Without loss of generality suppose that the old variable x appears negated in c_0, c_1, \dots, c_{19} and appears not negated in $c'_0, c'_1, \dots, c'_{19}$. For each i , $0 \leq i \leq 19$ replace $\neg x$ in c_i (respect. x in c'_i) with $\neg x_i$ (respect. x_i). Call the resulting formula Φ''' . In Φ''' each variable appears four times in the formula, twice negated and twice not negated. It is easy to see that Φ'' has a satisfying assignment if and only if Φ''' has a satisfying assignment.

Step 3.

Let $\mathcal{A} = \{A_i : i \in I\}$ be a finite family of (not necessarily distinct) subsets of a finite set \mathcal{U} . A system of distinct representatives (SDR) for the family \mathcal{A} is a set $\{a_i : i \in I\}$ of distinct elements of \mathcal{U} such that $a_i \in A_i$ for all $i \in I$. Hall's Theorem says that \mathcal{A} has a system of distinct representatives if and only if $|\cup_{i \in J} A_i| \geq |J|$ for all subsets J of I [15].

Let Φ''' be a given formula with the set of variables X and the set of clauses C . Let $\mathcal{U} = \{x, \neg x : x \in X\}$ and for every clause $c_i \in C$, $A_i = \{z : z \in c_i\}$. In Φ''' each variable appears four times in the formula, twice negated and twice not negated. Consider any union of k of the sets A_i . Since each A_i contains at least 2 distinct elements and no literal is contained in more than 2 sets, the union contains at least k distinct elements. Therefore, by Hall's Theorem, there exists a system of distinct representatives of \mathcal{U} . For each clause c denote its representative literal by z^c . Note that there is a polynomial-time algorithm which either finds an SDR or shows that one cannot exist by finding a violation of Halls condition.

Step 4.

For every variable $x \in X$, put a copy complete bipartite graph $K_{r-1, r-1}$ with the vertex set $[\mathcal{X}, \mathcal{Y}]$, where $\mathcal{X} = \{x_i | i \in [r-1]\}$ and $\mathcal{Y} = \{\neg x_i | i \in [r-1]\}$. Also, for every clause $c = (x \vee y \vee w)$ put the vertex c . Join the vertex c to one of the vertices x_1 or x_2 ; also, join the vertex c to one of the vertices y_1 or y_2 and join the vertex c to one of the vertices w_1 or w_2 , such that in the resulting graph $\max\{d(x_1), d(x_2), d(y_1), d(y_2), d(w_1), d(w_2)\} \leq r$.

Next, for every clause c join the vertex c to the vertices $z_3^c, z_4^c, \dots, z_{r-1}^c$. Call the resulting graph G . In G the degree of each clause vertex c is r and $\delta(G) \geq r-1$.

Step 5.

Let K_{r+1} be a complete graph with the vertices v_1, v_2, \dots, v_{r+1} . Let $e = v_1 v_2$ and $H_r = K_{r+1} \setminus \{e\}$. Consider two copies of G . For every vertex $v \in V(G)$ with $d(v) < r$, put a copy of H_r and join the vertex v_1 to the vertex v of the first copy of the graph G , and join the vertex v_2 to the vertex v of the second copy of the graph G . Call the resulting r -regular graph G' .

In the following we introduce the members of maximal independent set T .

Members of T .

Step 1.

For every subgraph G of G' put the set of vertices $\{c : c \in C\}$ in T .

Step 2.

For every subgraph H_r of G' put the vertices v_1 and v_2 in T . (H_r is introduced in Step 5, in the construction of G').

Let D be an independent dominating set for T and suppose that $c = (x \vee y \vee w)$ is an arbitrary clause. Without loss of generality suppose that $cx_1, cy_1, cw_1 \in E(G')$. By the structure of G' at least one of the vertices of the set $\{x_1, y_1, w_1, z_3^c, z_4^c, \dots, z_{r-1}^c\}$ is in D . On the other hand, for every variable $x \in X$, since we put a copy of complete bipartite graph $K_{r-1, r-1}$ with the vertex set $[\mathcal{X}, \mathcal{Y}]$, where $\mathcal{X} = \{x_i | i \in [r-1]\}$ and $\mathcal{Y} = \{\neg x_i | i \in [r-1]\}$ in G' . Therefore, if D contains a vertex from \mathcal{X} , then it does not have any vertex from \mathcal{Y} and vice versa. First, suppose that G' has independent dominating set D for T . Let $\Gamma : X \rightarrow \{true, false\}$ be a function such that $\Gamma(x) = true$ if and only if at least one of the vertices x_1, \dots, x_{r-1} is in D .

It is easy to see that Γ is a satisfying assignment for Φ'' . Next, let $\Gamma : X \rightarrow \{true, false\}$ be a satisfying assignment for Φ'' . For every x , put x_1, x_2 in D if and only if $\Gamma(x) = true$. It is easy to extend this set into an independent dominating set for T . \square

Proof of Theorem 2. (i) First, we introduce the construction of \mathcal{J}_k . For a given k -regular graph G , define:

$$\begin{aligned} V(\mathcal{J}_k) &= \{v_e | v \in V(G), e \text{ is incident with } v\} \cup \{e_{v,u,i} | vu = e \in E(G), i \in [k-1]\}. \\ E(\mathcal{J}_k) &= \{v_e v_{e'} | v \in V(G), e, e' \text{ are incident with } v\} \cup \{e_{v,u,i} e_{v,u,i'} | i \neq i'\} \cup \{v_e e_{v,u,i} | i \in [k-1], e \text{ is incident with } v\}. \end{aligned}$$

Now, consider the following useful lemma which will be used in our proof.

Lemma: Let G be a k -regular graph. There is function $f : V \times E \rightarrow [k]$ such that:

1. For every edge $e = uv$ in $E(G)$, $f(v, e) \neq f(u, e)$.
2. For each vertex v in $V(G)$, for every two edges e and e' incident with v , $f(v, e) \neq f(v, e')$.

Proof of Lemma : Consider the bipartite graph $G^{\frac{1}{2}}$ ($G^{\frac{1}{2}}$ is obtained from G by replacing each edge with a path with exactly one inner vertex). Since for every bipartite graph H , $\chi'(H) = \Delta(H)$ (see for example [15]). Therefore $\chi'(G^{\frac{1}{2}}) = k$. Consequently, there is function $f : V \times E \rightarrow [k]$. \square

Partitioning the vertices of a graph into t independent dominating sets is equivalent to a t -labeling of the vertices such that each vertex has no neighbors labeled the same as itself and at least one neighbor labeled with each of the other $t - 1$ labels.

Let \mathcal{J}_k be a graph which is constructed from G and f be a function $f : (v, e) \rightarrow [k]$ such that for every edge $e = uv$ in $E(G)$, $f(v, e) \neq f(u, e)$ and for each vertex v in $V(G)$, for every two edges e and e' incident with v , $f(v, e) \neq f(v, e')$. Consider the following labeling for the vertices of \mathcal{J}_k :

$$\ell(v_e) = f(v, e), \ell(e_{v,u,1}) = k + 1.$$

For every $e = uv$ define $\ell(e_{v,u,2}), \dots, \ell(e_{v,u,k-1})$ such that

$$\{\ell(v_e), \ell(u_e), \ell(e_{v,u,1}), \dots, \ell(e_{v,u,k-1})\} = [k + 1].$$

It is easy to see that ℓ is a $(k + 1)$ -labeling of the vertices such that each vertex has no neighbors labeled the same as itself and at least one neighbor labeled with each of the other k labels. So, the vertices of \mathcal{J}_k can be partitioned into $k + 1$ independent dominating sets.

(ii) It was shown that 3-colorability of planar 4-regular graphs is NP-complete [3]. For a given 4-regular graph G with n vertices we construct a graph \mathcal{H} with degree set $\{3n, 7n\}$ such that the vertices of \mathcal{H} can be partitioned into independent dominating sets if and only if G is 3-colorable. Define:

$$\begin{aligned} V(\mathcal{H}) &= \{v_j^k | j \in [n], k \in [3], v \in V(G)\} \cup \{v' | v \in V(G)\}, \\ E(\mathcal{H}) &= \{v'v_j^k | j \in [n], k \in [3], v \in V(G)\} \\ &\cup \{v_j^k v_{j'}^{k'} | j, j' \in [n], k, k' \in [3], (j, k) \neq (j', k'), v \in V(G)\} \\ &\cup \{u_j^k v_{j'}^{k'} | j, j' \in [n], k \in [3], vu \in E(G)\}. \end{aligned}$$

First, suppose that G is 3-colorable and let $f : V(G) \rightarrow \{0, 1, 2\}$ be a proper vertex coloring. Consider the following partition for the vertices of \mathcal{H} :

$$P = \{v' | v \in V(G)\},$$

$$P_\ell^h = \{v_j^k | j = \ell, f(v) = (h + k \bmod 3)\},$$

where $1 \leq \ell \leq n$, $0 \leq h \leq 2$. It is easy to see that these sets are disjoint independent dominating sets for \mathcal{H} and a partition for the vertices of \mathcal{H} .

Now, assume that G is not 3-colorable. Let $\mathcal{R} = \{v_j^k | j \in [n], k \in [3], v \in V(G)\}$ and $\mathcal{S} = \{v' | v \in V(G)\}$. To the contrary suppose that T_1, T_2, \dots, T_z is a partition of the vertices of \mathcal{H} and each T_i is an independent dominating set for \mathcal{H} . Consider the following partition for the vertices of \mathcal{H} :

$$V(\mathcal{H}) = \bigcup_{v \in V(G)} \mathcal{A}_v.$$

$$\mathcal{A}_v = \{v', v_j^k | j \in [n], k \in [3]\}.$$

By the structure of \mathcal{H} for every independent dominating set T_i and $v \in V(G)$ we have $|T_i \cap \mathcal{A}_v| = 1$. Therefore, for each T_i , $|T_i \cap \mathcal{R}| \leq n$, so $z \geq |\mathcal{R}|/n = 3n^2/n$. Therefore $z \geq 3n$. On the other hand, since G is not 3-colorable, therefore for every independent dominating set T_i , we have $|T_i \cap \mathcal{S}| \geq 1$. Consequently $|\mathcal{S}| \geq 3n$. But this is a contradiction. This completes the proof.

(iii) Given a graph G , a subset A of its vertex set is a 1-perfect code if A is an independent set and every vertex not in A is at distance one from exactly one vertex of A . In other words:

$$C \subseteq V(G) \text{ is 1-perfect code} \Leftrightarrow (\forall v \in V(G))(\exists! c \in C) d(v, c) \leq 1$$

It was shown that for a given 3-regular graph G determining whether the vertices of G can be partitioned into 1-perfect codes is **NP**-complete [6]. Note that every 1-perfect code in a 3-regular graph on n vertices has size $n/4$. So the vertices of a given 3-regular graph G can be partitioned into 1-perfect codes if and only if the vertices of G can be assigned 4 different colors in such a way that closed neighborhood of each vertex is assigned all 4 colors, i.e., $\chi(G^2) = 4$ (The square of a graph G , denoted by G^2 , is the graph obtained from G by adding a new edge joining each pair of vertices at distance 2). Therefore, from [6] we have the following corollary:

Corollary 1 *For a given 3-regular graph G determining whether $\chi(G^2) = 4$ is **NP**-complete.*

Let G be a 3-regular graph. Let $H = G \cup K_4$. Then the vertices of G can be partitioned into 1-perfect codes if and only if the vertices of H can be partitioned into independent dominating sets. This completes the proof. \square

Proof of Theorem 3. A *strong vertex coloring* of graph G is a proper vertex coloring of G such that for any $u, w \in N_G(v)$, u and w are assigned distinct colors. If $c : V(G) \rightarrow S$ is a strong vertex coloring of G and $|S| = k$, then G is called *k-strong-vertex colorable* and c is a *k-strong-vertex coloring* of G , where S is a color set.

It was shown that for a given graph G whose vertices have degree equal to k or 1 is $(k + 1)$ -incidence colorable if and only if G is $(k + 1)$ -strong-vertex colorable [8]. Since

for a given 3-regular graph G determining whether $\chi(G^2) = 4$ is **NP**-complete (for more details, see part 2 in the proof of Theorem 2), thus for a given 3-regular graph G deciding whether G is 4-incidence colorable is **NP**-complete. \square

3 Conclusion and Future Works

In this work, we proved that deciding the satisfiability of 3-SAT with the property that every literal appears in exactly 4 clauses (2 time negated and 2 times not negated), is **NP**-complete. We called this problem (2/2/3)-SAT. Note that if we consider 3-SAT problem with the property that every literal appears in 2 clauses (1 time positive and 1 time negative), then the problem is always satisfiable. Also, Tovey in [13] showed that instances of 3-SAT in which every variable occurs three times are always satisfiable. Therefore our result is sharp. In other words, in terms of (k, t, t') it is the tightest possible restriction in the sense that k -SAT remains **NP**-complete where each variable appears t time positive and t' time negative. Note that it is well known that 2-SAT can be solved in polynomial time. As an application of (2/2/3)-SAT problem, we proved that for each $r \geq 3$, the following problem is **NP**-complete: "Given (G, T) , where G is an r -regular graph and T is a maximal independent set of G , determine whether there is an independent dominating set D for T such that $T \cap D = \emptyset$ ". Regarding this result, solving the following question can be interesting.

- Determine the computational complexity of deciding whether a given regular graph has two disjoint independent dominating sets.

We proved that determine whether the vertices of a given 3-regular graph can be partitioned into independent dominating sets is **NP**-complete. However, one further step does not seem trivial.

- Determine the computational complexity of deciding whether the vertices of a given connected regular graph can be partitioned into independent dominating sets.

In [1], it was proved that if G is a non-empty graph, and T is an independent set of G , then there exists H such that, H is an independent dominating set for T and $\frac{|T \cap H|}{|T|} \leq \frac{2\Delta(G) - \delta(G)}{2\Delta(G)}$.

- Determine nontrivial upper bounds for the minimum cardinality of $|I \cap J|$ and also $\frac{|I \cap J|}{|T|}$ among all two independent dominating sets I and J of a graph G for some important family of graphs such as regular graphs.

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